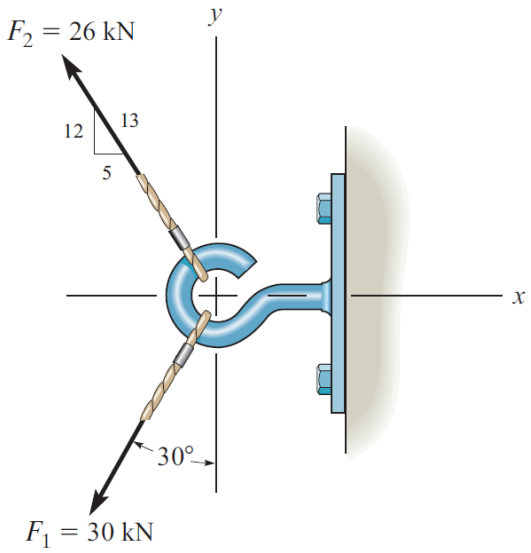
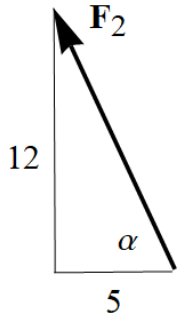
Problem 2-54

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Solution

Begin by finding the angle that $\mathbf{F}_2$ makes with the x-axis.



$$\tan\alpha = \frac{12}{5} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{12}{5}\right) \approx 67.4^\circ$$

Write each of the forces in component form.

$$\mathbf{F}_1 = 30\,\langle -\sin 30^\circ, -\cos 30^\circ \rangle \text{ kN}$$

$$\mathbf{F}_2 = 26\,\langle -\cos\alpha, \sin\alpha \rangle \text{ kN} = 26\left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle \text{ kN} = \langle -10.0, 24.0 \rangle \text{ kN}$$

Add them together to get the resultant.

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ &= \langle -30\sin 30^\circ - 10.0, -30\cos 30^\circ + 24\rangle \text{ kN} \\ &\approx \langle -25, -1.98\rangle \text{ kN}\end{aligned}$$

Its magnitude is

$$\begin{aligned}|\mathbf{F}_R| &= \sqrt{(-30\sin 30^\circ - 10.0)^2 + (-30\cos 30^\circ + 24)^2} \\ &\approx 25.1 \text{ kN},\end{aligned}$$

and the direction it points in counterclockwise from the positive x-axis is

$$\tan\theta = \frac{-30\cos 30^\circ + 24}{-30\sin 30^\circ - 10.0} \quad \rightarrow \quad \theta \approx 180^\circ + \tan^{-1}\left(\frac{-1.98}{-25}\right) \approx 185^\circ.$$